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Section 10. Dielectric methods, theory and simulation

Is there geometrical/physical meaning of the fractional integral with complex exponent?

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Abstract

The geometrical/physical meaning of the temporal fractional integral with complex fractional exponent has been found and discussed. It has been shown that the imaginary part of the fractional integral is related to *discrete scale invariance* (DSI) phenomenon and observed only for *true* regular (discrete) fractals. Numerical experiments show that the imaginary part of the complex fractional exponent can be well approximated by a simple and finite combination of the leading sine/cosine log-periodical functions with period $\ln \xi$ (ξ is a scaling parameter). In most cases analyzed, the leading Fourier components give a pair of complex conjugated exponents defining the imaginary part of the complex fractional integral. For random fractals, where invariant scaling properties are realized only in the statistical sense the imaginary part of the complex exponent is *averaged* and the result is expressed in the form of the conventional Riemann–Liouville integral. The conditions for realization of reind and recaps elements with *complex power-law exponents* have been found. Description of relaxation processes by kinetic equations containing complex fractional exponent and their possible recognition in the dielectric spectroscopy is discussed. New kinetics expressed in terms of non-integer operators with complex and real power-law exponents can be successfully applied for description of dielectric spectra of many non-crystalline solids.

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1. Introduction

Recently much attention has been paid to existence of equations containing *real* fractional exponent [1–3]. It becomes evident that equations with fractional derivatives can play a crucial role in description of kinetic and transfer phenomena in mesoscale region. As it was already discussed in paper [4] the frontiers of science are rapidly shifting from the investigation of the basic bricks of matter to the elucidation of mesoscopic principles of its organization. Moving in this way we need a mathematical apparatus, which adequately corresponds to a true description of properties of a matter on mesoscale region. From our point of view this necessary mathematical instrument should lie in deep understating of the 'physics' of the fractional calculus. The first attempt to understand the result of averaging of a smooth function over the given fractal (Cantor) set has been undertaken in [1]. In the note [5] and later in paper [6] some doubts were raised to the reliability of the previously obtained result. The criticism expressed in these publications forced the author (RRN) to reconsider the former result, and the detailed study of this problem showed that the doubts had some grounds and were directly linked with the relatively delicate questions of averaging a smooth function over fractal sets, in particular, on Cantor set and its generalizations. But we cannot agree with final conclusion made in [6]: 'no direct

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